COEFFICIENTS CHARACTERIZING THE RESPONSE OF A POSISTOR RESISTANCE TO CHANGES IN HEAT-TRANSFER CONDITIONS

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The temperature and power coefficients of the resistance of a posistor are examined together with the relative power response.

In connection with steady-state and non-steady-state processes in circuits that contain temperature-dependent resistances, an important role is played by the coefficients characterizing the response of the thermistor resistance to changes in heat-transfer conditions. The heat-transfer conditions determine the temperature of the thermistor T and the dissipated power P_α.

In thermistor theory, the following coefficients are considered:

1) the temperature coefficient α_t of the electrical resistance which characterizes the response of the thermistor resistance to changes in the temperature of the thermistor:

$$\alpha_t = -\frac{1}{R_t}\frac{dR_t}{dT}; \qquad (1)$$

2) the power (energy) coefficient γ_T of the electrical resistance, which characterizes the response of the thermistor resistance to changes in dissipated power:

$$\gamma_t = -\frac{1}{R_t}\frac{dR_t}{dP_\alpha}; \qquad (2)$$

3) the dynamic coefficient D or relative power response [1]

$$D = \gamma_t P_\alpha. \qquad (3)$$

The posistor is also characterized by these coefficients.

Our problem is to investigate the coefficients α_p, γ_p, and D for the posistor.

Temperature coefficient of posistor resistance α_p. For thermistors, the temperature coefficient α_t is negative and depends only on the temperature of the thermistor. Owing to its characteristic varistor effect, the posistor behaves differently. When the posistor is heated through 1° C by the ambient medium or by the current flowing through it, the change of resistance is not the same. The reason for this is as follows. As the temperature of the ambient medium varies at $P_\alpha \approx \approx 0$, the resistance of the posistor is affected only by its temperature.

For this case, we can write

$$\alpha_{po}(T) = \frac{1}{R_{po}(T)}\frac{dR_{po}(T)}{dT} = \frac{d\ln R_{po}(T)}{dT}. \qquad (4)$$

The coefficient $\alpha_{p0}(T)$ is determined by graphic differentiation of the temperature characteristic $\ln R_{p0}(T) = f(T)$.

When the posistor is heated by the current flowing through it at T_0 = const, a more complicated effect is observed. In this case the resistance of the posistor changes not only as a result of the change in its temperature (thermal effect) but also as a result of the change in the input voltage (varistor effect).

In this case, we have

$$\alpha_p(T, U) = \frac{1}{R_p(T,U)}\frac{dR_p(T, U)}{dT} = \frac{d\ln R_p(T, U)}{dT}. \qquad (5)$$

We now show that there is a regular relationship between the coefficients $\alpha_{p0}(T)$ and $\alpha_p(T, U)$.

The resistance of the posistor is given by the expression

$$R_p(T, U) = R_{po}(T)\exp\left[-b(T)(\sqrt{U}-1)\right]. \qquad (6)$$

The nonlinearity factor b is a function of the temperature of the posistor, the relation $b = f(T)$ resembling the temperature characteristic of the posistor to a semilogarithmic scale.

We take logarithms on both sides of expression (6)

$$\ln R_p(T, U) = \ln R_{po}(T) - b(T)(\sqrt{U}-1), \qquad (7)$$

then differentiate expression (7) with respect to dT at U = const

$$\frac{d\ln R_p(T, U)}{dT} = \frac{d\ln R_{po}(T)}{dT} - \frac{db(T)}{dT}(\sqrt{U}-1)$$

or

$$\alpha_p(T, U) = \alpha_{po}(T) - \beta(T)(\sqrt{U}-1), \qquad (8)$$

where

$$\beta(T) = \frac{db(T)}{dT}. \qquad (9)$$

Equation (8) reflects the dual nature of the posistor. The properties of the posistor regarded as a heat-dependent element are reflected by the term $\alpha_{p0}(T)$ associated with the thermal effect; this is determined by graphic differentiation of the temperature characteristic $\ln R_{p0} = f(T)$. The varistor properties of the posistor are reflected by the term $\beta(T)$; this is determined by the varistor effect and is found by graphic differentiation of the curve $b = f(T)$. The minus sign in expression (8) is a consequence of the mutual op-

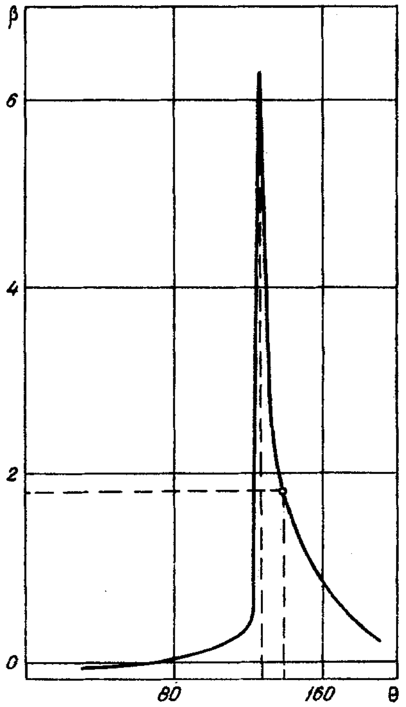

Fig. 1. The coefficient β, $V^{-1/2}$/deg, as a function of the posistor temperature Θ, °C.

position of these effects. In fact, on the temperature interval 70–190° C the temperature coefficient α_{p0} is positive, since the relation $\ln R_{p0} = f(T)$ is an increasing function. The second term on the right-hand side of (8) determines the varistor properties of the posistor. As the voltage increases, the resistance of the varistor decreases. Consequently, the nonlinearity factor b is a negative quantity. It is clear from (8) that the higher the voltage applied to the posistor, the more strongly the coefficients α_p and α_{p0} differ. In

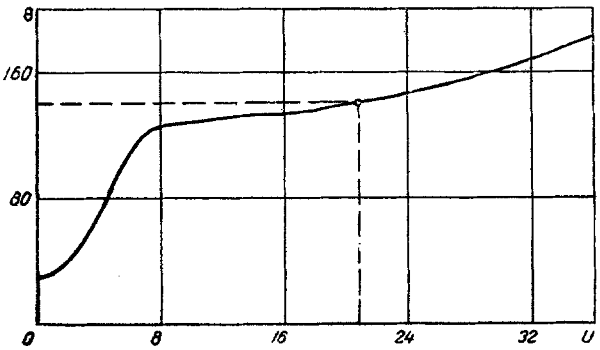

Fig. 2. Posistor temperature Θ, °C, as a function of the applied voltage U, in V, at an ambient temperature Θ_0 = 70° C.

particular, when the applied voltage is several tens of millivolts, the second term in expression (8) can be neglected and then $\alpha_p = \alpha_{p0}$.

The curve $\beta = db/d\Theta = f(\Theta)$ in Fig. 1 recalls the relation $\alpha_{p0} = f(\Theta)$ presented in [2, 3].

From the above there follows a method of determining the temperature coefficient α_p at any point on the static current-voltage characteristic. By assigning different values of the posistor temperature Θ, from the experimentally constructed $\Theta = f(U)$ curve for Θ_0 = const we find the voltage values corresponding to these temperatures (Fig. 2).

From the relations $\alpha_{p0} = f(\Theta)$ and $\beta = f(\Theta)$ we find the values of the coefficients α_{p0} and β for the same temperatures (Figs. 1 and 2). Substituting the values of the corresponding parameters into expression (8), we find the value of the temperature coefficient α_p at the points on the static current-voltage characteristic corresponding to the voltages obtained.

Power coefficient γ_p of the posistor resistance. The power coefficient γ_p can be determined from the

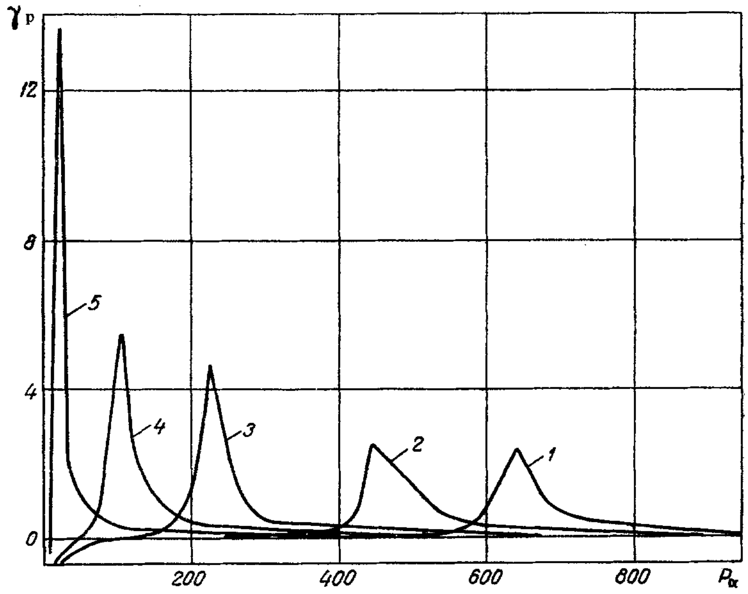

Fig. 3. Coefficient γ_p, %/mW, as a function of the dissipated power P_α, mW, with the ambient temperature as parameter: 1) Θ_0 = 18° C; 2) 50; 3) 90; 4) 110; 5) 125.

expression

$$\gamma_p = \frac{\alpha_p}{k} = \frac{\alpha_{po}(T) - \beta(T)(\sqrt{U} - 1)}{k}, \quad (10)$$

where k is the posistor dissipation factor. Moreover, the power coefficient can be determined from the $\ln R_p = f(P_\alpha)$ curves constructed with the temperature of the ambient medium as parameter. Since

$$\gamma_p = \frac{1}{R_p}\frac{dR_p}{dP_\alpha} = \frac{d \ln R_p}{dP_\alpha} \approx \frac{\Delta \ln R_p}{\Delta P_\alpha}, \quad (11)$$

having graphically differentiated these curves with respect to P_α, we find the change in the coefficient γ_p along the static current-voltage characteristic. Analysis showed that up to an ambient temperature equal to the Curie temperature the $\gamma_p = f(P_\alpha)$ curves have a peak and resemble the shape of the $\alpha_{p0} = f(\Theta)$ curve (Fig. 3). As the ambient temperature increases, the magnitude of the γ_p peak steadily grows (from 2.145%/mW at Θ_0 = 18° C to 19.2%/mW at Θ_0 = 127° C).

At ambient temperatures above the Curie point, the coefficient γ_p is negative, and the $\gamma_p = f(P_\alpha)$ curves do not have a peak. At Θ_0 in the range 150–200° C, the power coefficient reaches very large values at small dissipated powers. The higher the ambient temperature, the higher the value of γ_{pmax}, which at Θ_0 = 190° C reaches 350%/mW.

Relative power response—the coefficient D. Singular point on the static current-voltage characteristic. The coefficient D may be called the relative power response [1]

$$D = \frac{dR_p/R_p}{dP_\alpha/P_\alpha}. \quad (12)$$

The variation of D along the static current-voltage characteristic is represented in Fig. 4 for Θ_0 = 70° C.

As may be seen from Fig. 4, at ambient temperatures not exceeding the Curie temperature, the $D = f(U)$ curve has a fairly complicated shape, passing twice through zero and unity, and having a maximum and negative values.

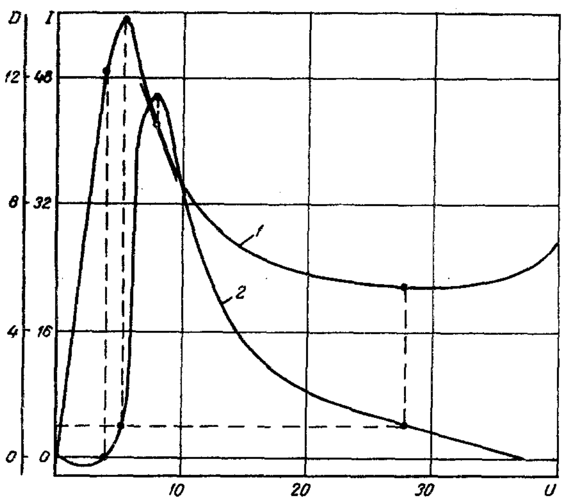

Fig. 4. Static current-voltage characteristic of posistor at Θ_0 = 70° C (curve 1) and the $D = f(U)$ relation for this characteristic (curve 2). (D in dimensionless units; I in mA; U in V.)

The coefficient D passes through unity at the maximum and minimum points of the static current-voltage characteristic, which can be proved starting from Eq. (12). In view of the fact that

$$R_p = \frac{U}{I} \quad dR_p = d\left(\frac{U}{I}\right) = \frac{IdU - UdI}{I^2},$$

$$P_\alpha = UI \quad dP_\alpha = d(UI) = UdI + IdU,$$

$$D = \frac{\frac{I}{U}\,\frac{IdU - UdI}{I^2}}{\frac{1}{UI}(UdI + IdU)} = \frac{I/U - dI/dU}{I/U + dI/dU}. \quad (13)$$

Since at the maximum and minimum of the static current-voltage characteristic $dI/dU = 0$, the expression

$$D = \frac{I/U - dI/dU}{I/U + dI/dU} = 1. \quad (14)$$

For the points at which the coefficients γ_p, α_p, and D are equal to zero

$$D = \frac{I_0/U_0 - dI/dU}{I_0/U_0 - dI/dU} = 0. \quad (15)$$

For this expression to be equal to zero, it is necessary and sufficient that

$$I_0/U_0 - dI/dU = 0,$$

$$I_0/U_0 = dI/dU,$$

or

$$\sigma_{p,st} = \sigma_{p.dyn};$$

i. e., at these points the static and dynamic conductances (and hence resistances) are equal.

Up to this point, the current-voltage characteristic is approximately a straight line, but beyond it, it is distinctly curved.

To the left of this point, $D < 0$; consequently,

$$\sigma_{p.st} < \sigma_{p.dyn},$$

the dynamic conductance being positive on this interval.

On the interval $0 < D < 1$

$$\sigma_{p.st} > \sigma_{p.dyn},$$

and on this interval the dynamic conductance is also positive, whereas at $D > 1$ it becomes negative.

Along the static current-voltage characteristic the maximum of the coefficient D coincides with the maxima of α_p and γ_p. Moreover, at these same points the $\Theta = f(U)$ curves have an inflection (Fig. 2) and the linear $\Theta = f(P_\alpha)$ relations for various ambient temperatures show a discontinuity.

Consequently, these are singular points of the static current-voltage characteristics characterized by maximum values of the coefficients α_p, γ_p, and D. At these points on the static current-voltage characteristics, the temperature of the posistor is equal to the Curie temperature.

For D at the maximum, taking into account that here the differential conductance is negative, we have

$$D=\frac{\sigma_{p.st}-(-\sigma_{p.dyn})}{\sigma_{p.st}+(-\sigma_{p.dyn})}=\frac{\sigma_{p.st}+\sigma_{p.dyn}}{\sigma_{p.st}-\sigma_{p.dyn}}=D_{max}. \quad (16)$$

For this fraction to have a maximum value, it is sufficient that

$$\sigma_{p.dyn}=dI/dU=(\sigma_{p.dyn})_{max},$$

i. e., the point of maximum of the relation $D = f(U)$ coincides with the point of maximum negative differential conductance on the static current-voltage characteristic. The same point will be the point of minimum negative differential resistance.

Since at these points dI/dU = max,

$$d^2I/dU^2=0.$$

Such points are called linearization points, since at them the curve approaches a straight line, which can also be seen by examining the static current-voltage characteristic (Fig. 4). At the linearization points the curve is particularly close to its tangent.

Thus, having the static current-voltage characteristic of the posistor, we can graphically determine the linearization point. It will be located in the middle of the interval on which the $I = f(U)$ curve is closest to its tangent. This point will also be the singular point of the current-voltage characteristic whose properties were enumerated above.

At high ambient temperatures the coefficient D varies little, fluctuating about some small constant value.

The results obtained can be used in analyzing steady-state and dynamic modes in circuits containing posistors when the temperature of the ambient medium and the dissipated power vary.

NOTATION

R_t is the thermistor resistance; α_t and γ_t are the temperature and power coefficients of the thermistor resistance; R_{p0} is the resistance of the posistor determined from its temperature characteristic; R_p is the resistance of the posistor determined from its static current-voltage characteristic; α_{p0} is the temperature coefficient of the posistor resistance determined from its temperature characteristic; α_p is the temperature coefficient of the posistor resistance for heating by the current at constant ambient temperature; γ_p is the power coefficient of posistor resistance; T and Θ are the posistor temperature in °K and °C, respectively; T_0 and Θ_0 are the ambient temperature in °K and °C, respectively; D is the relative power response; U is the voltage; I is the current; P_α is the dissipated power; k is the dissipation factor; b is the coefficient of nonlinearity of the varistor characteristic; $\beta = db/dT$; $\sigma_{p.st}$ is the static conductance; $\sigma_{p.dyn}$ is the dynamic conductance.

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